Industrial Image Processing

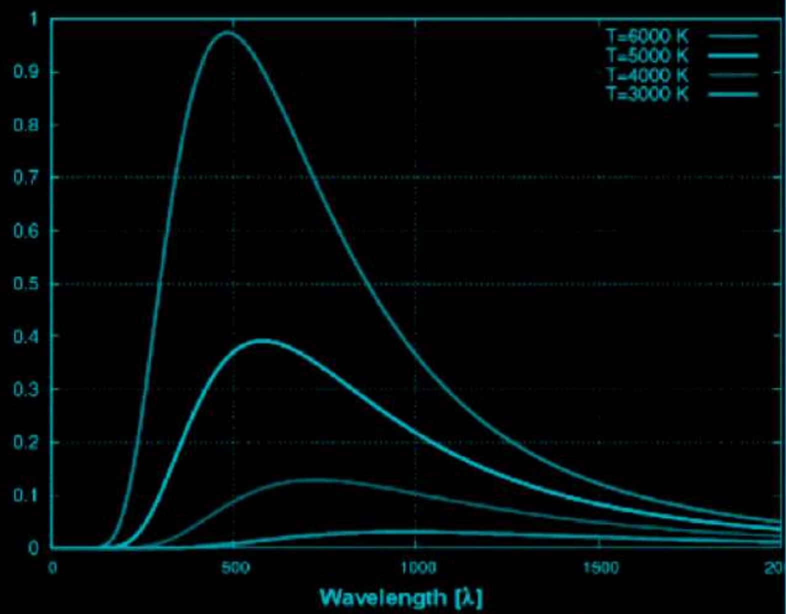
Optics

$$E = \pi L$$

$$E_{obj} = \frac{\pi}{80.000}\frac{(a^2 + y^2) \cdot N^4 \cdot ps^2 \cdot \delta z^2}{a^4\, S \cdot y^4 \cdot t}$$

$$\Phi_e[W] = FWHM[nm] \cdot 0.02 \quad @\, 6000\,K,\ 1000\,lm$$

$$\Phi_e[W] = (\lambda[\mu m] - 0.4)\frac{FWHM[nm]}{10} \quad @\, 3000\,K$$



Lamp	CRI
Halogen	100
White LED	80...95
Sodium	24
White Fluorescence	50 ... 80

Tobias Haist

Foreword

Industrial image processing is a mixture of optics, software engineering, automation and production technology. This document (and of course the wallpaper) is about the often neglected part of industrial image processing: optics. Based on my experience, most people working in the field do not originally have a background in optics. For those people it is sometimes hard to decide about the hardware of the system. This then might result in poor image quality (or excessive cost). Digital processing of the images then becomes unnecessarily difficult or even impossible, e.g. if small defects are just not resolved.

Even if you studied optics, there is often a lack of knowledge about the practical decisions necessary to implement a system. In principle, everything is simple, but in practice you have to consider a lot of things. Since in most cases the right choice of the illumination and the decisions concerning "resolutions" are difficult, I will put some emphasis on that.

This document is not a replacement of an optics course or an optics textbook. It is meant to show you how to use the equations on the wallpaper. I will give you a short description of every equation and simple examples showing their usage. Section 2 deals with some more extended typical problems that, hopefully, show you how to use the equations in combination.

Most equations can be found in a lot of different books. I tried to limit most of the references to three easily available and excellent sources [1, 2, 3]. A few more complex equations are explained in the appendix. Let me clarify the aim of the selection of the equations: They should help you to do some *practical engineering*. I have selected the equations purely based on their importance for practical applications in the field.

Of course, this reflects a subjective judgement and you might want to add some other equations that are especially important for your work. You also might want to use different background images. Please feel free to do so. For simple graphical changes, I recommend to use gimp (use the accompanied xcf file). For adding (or correcting) equations, you can use the tex file (compile with latex, show with xdvi, make a highres screenshot, zoom and insert into the xcf).

There is no copyright whatsoever on these files. Use or change them the way you want and put them wherever you want (but it would be nice to cite me). Of course, all equations come without any warranty. If you find any errors: Please contact me, all help is welcome! Thanks in advance.

Just another short tip for using the equations: If you do some nontrivial engineering computations, it is always a good idea to use octave (or something similar) instead of a simple pocket calculator. To this end you just type the calculations in your favorite text editor, that is: you write a “script”, e.g. “example.m”. Then you run the script (“octave example.m”).

This offers three major advantages: 1) It is easy to rerun the computation with different numbers so you can easily check different scenarios. 2) You have documented your work and can easily give it to colleagues to check. 3) You can later easily correct for errors.

Alternatively, you might want to try Matlab, Mathematica or something similar. But typically these tools are quite large and, therefore, not so fast (and of course not free). So give octave a try. You might love it.

**Stuttgart, March 2014, Tobias Haist (haist@ito.uni-stuttgart.de),
Version V 1.0.**

1 Equations

1.1 Imaging

For conventional imaging you might have a look at [1]. For much more details and lots of examples see [4]. A lot of details can be also found in [5].

To avoid unnecessary mistakes, it is mandatory to use the correct sign convention. In this book, the coordinate system has its origin at the center of the (thin) imaging lens. This means that the object distance and the magnification are typically negative ! Variables with primes, e.g. f', denote quanities in image space, whereas unprimed variables are in object space.

1.1.1 Magnification

$$\boxed{\beta' = \mathbf{M} := \frac{\mathbf{y'}}{\mathbf{y}} = \frac{\mathbf{a'}}{\mathbf{a}}}$$

β' - magnification
y' - image height
y - object height
a - object distance (approximately working distance)
a' - image distance

Example: We want to achieve a magnification of -0.1 (beware: in most situations the sign of the magnification is negative because the image is inverted) and the object distance is 1 m (again: due to the sign convention, a is typically negative). Compute the image distance.

Solution: $\beta' = -0.1 = a'/a \ \rightarrow a' = 0.1$ m

See W.J. Smith: "Modern Optical Engineering", SPIE Press , chapter 2

1.1.2 Image and object distance

$$\boxed{\frac{1}{a'} = \frac{1}{a} + \frac{1}{f'}}$$

a - object distance (approximately working distance)
a' - image distance
f' - focal length

Example: Compute the focal length to achieve a magnification of -0.5 at an object distance of 1 m (a = -1 m).

Solution: Due to $\beta' = a'/a = -0.5$ we get $a' = 0.5$. Now we can compute $1/f' = 1/a' - 1/a$ and it follows $f' = 0.33$ m.

See W.J. Smith: "Modern Optical Engineering", SPIE Press , chapter 2

1.1.3 Object distance

$$\boxed{\mathbf{a} = \mathbf{f}'\,(\frac{\mathbf{1}}{\beta'} - \mathbf{1})}$$

a - object distance (approximately working distance)

β' - magnification

f' - focal length

In principle this equation is not necessary because we can derive it easily from the other equations. Anyway, since we often need it, here it is.

Example: Compute the object distance given a focal length of 100 mm if the magnification is -0.1.

Solution: $a =$ 100 mm (-1 - 10) $=$ -1100 mm

1.1.4 F-number

$$\boxed{F\# = K = \frac{f'}{D_{EP}} = \frac{1}{2\,NA'}}$$

$F\#$ - F-number
f - focal length
D_{EP} - diameter of the entrance pupil
NA' - image-sided numerical aperture

Example: Compute the F-number of a lens with focal length 100 mm and a clear diameter of 10 mm.

Solution: $K = 100/10 = 10$

See W.J. Smith: “Modern Optical Engineering”, SPIE Press , chapter 6

1.1.5 Effective F-number

$$\boxed{\mathbf{K_{eff}} := (\mathbf{1} + \beta')\mathbf{K}}$$

K_{eff} - effective F-number

β' - magnification

K - F-number

The F-number is (strictly) defined for the situation that the object is located at infinity. For objects at finite distances with a certain magnification β', which is the conventional case in industrial image processing, sometimes the effective F-number is used. Most of the time, β' is quite small so that the $K_{eff} \approx K$.

1.1.6 Object-sided numerical aperture

$$\boxed{\mathrm{NA} = \mathrm{NA}' \, \beta'}$$

NA - object-sided numerical aperture ($n \sin \alpha$)
NA' - image-sided numerical aperture ($n' \sin \alpha'$)
β' - magnification

Example: You have a microscope with an objective lens 100x/0.8. Compute the numerical aperture of the light falling onto the CCD sensor.

Solution: 100x/0.8 means that the magnification is 100 and the numerical aperture in object space is 0.8. Therefore we can directly conclude that the numerical aperture in image space is given by $0.8/100 = 0.0008$.

See W.J. Smith: "Modern Optical Engineering", SPIE Press , chapter 6

1.1.7 Depth of field

$$\boxed{\Delta z' = \beta'^2 \, \Delta z}$$

$\Delta z'$ - image axial shift
Δz - object axial shift
β' - magnification

A shift in the object plane leads to a somewhat "amplified" or "deamplified" shift in the image plane.

Example: You have an imaging system that images an object with a diameter of 1 m onto a 1/3 inch CCD sensor. You are using a low F-number for imaging. Now the object is axially shifted by 100 mm. How much do you have to refocus the image plane to obtain again a sharp image?

Solution: The 1/3 inch sensor has a vertical extension of 3.6 mm (see table on the right-hand side of the wallpaper). Therefore $|\beta'| = 3.6/1000 = 0.0036$ and with that $\Delta z' = 1.3$ microns.

Compare that with the depth of field (next equation) and it becomes obvious (DOF is several meters) that indeed refocusing in this case would not be necessary.

See W.J. Smith: "Modern Optical Engineering", SPIE Press , chapter 2

1.1.8 Depth of field

$$\boxed{\delta \mathbf{z} \approx \mathbf{2K\,B'}/\beta'^{\mathbf{2}}}$$

δz - depth of field (in object space)

β' - magnification

K - F-number

B' - diameter of allowed blur circle in the image plane

Example: Compute the depth of field in object space for a an imaging system with $\beta' = -0.1$ and a F-number of 5. The allowed blurring is 10 μm (2 pixels).
Solution: $\delta z = 2 \cdot 5 \cdot 0.01\,\text{mm}/0.01 = 10$ mm

See W.J. Smith: "Modern Optical Engineering", SPIE Press , chapter 2

1.1.9 Depth of field - NA

$$\boxed{\delta \mathbf{z}' = \frac{\lambda}{2\mathrm{NA}'^2}}$$

$\delta z'$ - depth of field in image plane
NA' - image-sided numerical aperture
λ - wavelength of the light

This equation gives us the diffraction limited depth of field (Rayleigh's $\lambda/4$ peak-to-valley criterion). Within this limit you will barely see any difference compared to perfect focusing.

You can also use this equation for unprimed quantites in the object plane.

Example: You are using a 100x/0.8 microscope objective lens. What will be the depth of field with visible light (e.g. $\lambda = 500$ nm) ?

Solution: $\delta z = 500\,\mathrm{mm}/(2 \cdot 0.8^2) = 390$ nm

See R.E. Fisher, B. Tadic-Galeb, P.R. Yoder: "Optical System Design", SPIE Press , pg. 57

1.1.10 Front and back planes being in focus

$$\boxed{\mathbf{a}_{1/2} = \frac{\beta' \cdot \mathbf{a} \cdot \mathbf{D}_{\mathbf{EP}}}{\beta' \cdot \mathbf{D}_{\mathbf{EP}} \pm \mathbf{B}'}}$$

$a_{1/2}$ - front and rear object distances that limit the depth of field
a - object distance (in focus)
β' - magnification
D_{EP} - diameter of the entrance pupil
B' - diameter of allowed blur circle in the image plane

Example: We come back to the DOF case of section 1.1.8. Now we want to compute the near and far distance that is in focus. We assume that the nominal object distance where we focus onto is 1 m.

Compute the depth of field for a an imaging system with $\beta' = -0.1$ and an F-number of 5. The allowed blurring is 10 μm (2 pixels).

Solution: $\beta' = -0.1 = a'/a \approx f'/a = f'/1\,m$. Therefore $f' = 100$ mm. With that and K=5 we get $D_{EP} = 20$ mm. This leads to a_1= 1.005 m and $a_2 = 0.995$ m and — again — to a DOF of 10 mm.

Beware: The near and far distance are not always symmetric around the best focus plane.

1.1.11 Hyperfocal distance

$$\boxed{a_{Hyperf} \approx \frac{f'^2}{K \cdot B'}}$$

a_{Hyperf} - hyperfocal distance
f' - focal length
K - F-number
B' - diameter of allowed blur circle in the image plane

Example: Suppose you want to image a scene with an extension far into the distance with an F-number of 10 and a 50 mm lens. Where should you focus on and what is the nearest object that is still in focus? Assume a tolerable blur diameter of 3 pixels, each being 5 microns.

Solution: B' is 15 microns so we conclude

$$a_{hyperf} = \frac{50^2}{10 \times 0.015} \approx 16 \text{ m} \tag{1}$$

We have to focus on 16 m and our field will range from 8 m (half of the hyperfocal distance) to infinity.

See A. Hornberg, "Handbook of Machine Vision", Wiley, chapter 4

1.1.12 Telescope

$$\boxed{\beta\text{‘} = -\frac{\mathbf{f_2}}{\mathbf{f_1}}}$$

β' - magnification

f_1 - focal length of first lens

f_2 - focal length of second lens

The Kepler telescope is a standard and often used imaging geometry and achieves both-sided telecentricity provided that the diameters of the lenses are large enough. You can easily combine two standard objective lenses and build your own telecentric system and you can achieve quite high numerical apertures with moderate aberrations.

Example: Build a telecentric system with $\beta' = -0.1$. We use a quite small focal length (so save space) for the second lens: $f_2 = 10$ mm. Therefore, f_1 should be a 100 mm lens.

See W.J. Smith: "Modern Optical Engineering", SPIE Press , chapter 9

1.2 Resolution

1.2.1 Airy two-point separation

$$\boxed{\mathbf{r}' = \mathbf{0.61}\frac{\lambda}{\mathrm{NA}'} = \mathbf{1.22}\,\lambda\,\mathbf{K}}$$

r' - resolution according to Airy in the meaning of minimal point separation in the image plane. This is half of the diameter of the first minimum extension of the Airy pattern.
λ - wavelength of light
NA' - image sided numerical aperture
K - F-number

Example: You have a diffraction limited objective lens with the stop set to K=9. What size do you expect for the diameter of the image of an ideal point ?
Solution: $2r = 2.44\lambda K = 2.44 \cdot 500\text{nm} \cdot 9 = 10.9$ microns.

This equation can also easily be used for microscopic applications. Here, the "object" is to be regarded as the image. So if we want to compute the resolution of the 0.8 NA microscope objective, we directly get $r = 0.61 \cdot 500\text{nm}/0.8 \approx 380$ nm.

See R.E. Fisher, B. Tadic-Galeb, P.R. Yoder: "Optical System Design", SPIE Press , chapter 3 and Melles Griot Technical Guide, cvimellesgriot.com , Diffraction Effects

1.2.2 Critical Dimension

$$\boxed{|\beta'|\,\mathbf{CD} = \mathbf{Q} \cdot \mathbf{p}}$$

β' - magnification

CD - critical dimension in the object plane

Q - Q-factor (defined to be $Q = r'_{airy}/pixelsize$)

p - pixelsize

Example: You should design a spying satellite with Q=2 for a 10 microns imager that has a magnification of $\beta' = -1 \cdot 10^{-4}$. What is the resolution (two point separation) on the ground (object space) ?
Solution: $CD = 2 \cdot 10\mu m \cdot 10^4 = 0.2$ m

Please combine that with the table on the left-hand side of the wallpaper.

Example: You want to detect neighboring lines on an object. The minimum distance is 50 microns. The camera has pixels with a width of 5 microns. What can you say ?
Solution: According to the table "Neighbouring points or lines", we have something like $r_A = 0.8 \cdot 50 = 40$ microns on the object. We also need 3.5 Pixels for the 50 microns so we can say that one pixel should correspond (on the object side) to 14 microns. This means that we need a magnification of $\beta' = -14/50 =$ -0.28. Therefore, the Airy resolution in image space is $40 \cdot 0.28 = 11.2$ microns. This leads to an F-number of $K = 11.2/1.22 \cdot 500 = 18$. So, for the resolution we do not care much about having a small F-number. Of course, the aberrations of the lens have to be small enough to obtain the resolution (which should not be a problem at K=18).

See R.D. Fiete: "Modelling the Imaging Chain of Digital Cameras", SPIE Press

1.2.3 Maximum spatial frequency

$$\boxed{\mathbf{f'_{max} \approx \frac{1.22}{r'_{spot}} \approx \frac{1}{\lambda K}}}$$

f'_{max} - the limiting spatial frequency of the imaging system,

$MTF(f_{max}) = 0$ in the imaging plane

r'_{spot} - spot radius of the imaged spot in the image plane

λ - wavelength of light

K - F-number

Example: We found (see last example) that we need a spotradius of 11 microns in the image plane. Your supplier offers a lens that delivers a contrast of 30% at 60 lp/mm. Will that work ?

Solution: We first compute $f'_{max} = 1.22/11$ microns $= 110.000$ lp/m $= 110$ lp/mm.

Now we have a look at the lens being offered. According to

$$\boxed{\mathbf{MTF(f') = 0.3 \approx 1 - f'}}$$

with the normalized frequency f' (normalized of course to f'_{max}) for the 30% we have an $f' = 0.7$ for the **not normalized** spatial frequency. Therefore, for the unnormalized frequency at 30% contrast we can write $f'_r = 0.7 \cdot f'_{max} = 60$ lp/mm and therefore $f'_{max} = 60$lp/mm/0.7 $= 85$ lp/mm. Conclusion: the offered lens will not be sufficient.

See W.J. Smith: "Modern Optical Engineering", SPIE Press and Melles Griot Technical Guide, cvimelles-griot.com , Modulation Transfer Function

1.2.4 Spotsize due to spherical aberration

$$\boxed{\mathbf{r'_{SA} \approx 0.07 \cdot \frac{f'}{K^3}}}$$

r'_{SA} - spotsize due to spherical aberration for a single plano-convex lens
f' - focal length
K - F-number

Example: Estimate the spot size due to spherical aberration of a singlet compared to Airy resolution for f'=100 mm and $K = 7$.
Solution: $r_{SA} = 0.07 \cdot 100$ mm / $7^3 = 0.02$ mm $= 20$ microns. The radius due to Airy would be 4.2 microns.

Note: In practice, we typically buy achromats because the cost advantage of singlets is not so dramatic for small quantities. Also, the whole approach only makes sense for imaging near the optical axis (very small field size) because other aberrations will be present in addition off-axis.

See Melles Griot Technical Guide, cvimellesgriot.com Lens Selection Guide

1.2.5 Spotsize given encircled energy

$$\boxed{\mathbf{r'_{Spot}} \approx \frac{\sqrt{\mathbf{EncEng}}}{\mathbf{2}} \mathbf{5}\pi \mathbf{W_{RMS}} \lambda \mathbf{N} \cdot \mathbf{K}}$$

r'_{Spot} - radius of the spot that contains the given amount of encircled energy
$EncEng$ - amount of encircled energy (0..1)
W_{RMS} - RMS wavefront error in waves (lambdas)
λ - wavelength
N - number of bumps of wavefront (1 for defocus) K - F-number

Example: Assume that we have a lens with K=5 that has twice the permissible aberration due to Rayleigh to be called "diffraction-limited". The number of bumps in the wavefront is 3. Compute the spot radius where 80% of the energy is enclosed.
Solution: Rayleigh's $\lambda/4$ criterion says that we have diffraction limited performance if the peak-to-valley wavefront error is below $\lambda/4$. Now, a (very !) rough relation between PV and RMS error is: $PV \approx 6 \cdot RMS$ so the Rayleigh criterion gives us something like 0.08 λ RMS. Therefore, we obtain $r = \sqrt{0.8}/2 \cdot 5 \cdot \pi \cdot 0.08 \cdot 500 \cdot 10^{-9} \cdot 3 \cdot 5$ =4.2 microns.

According to Rayleigh, we would have a diffraction limited spotsize $r = 1.22\lambda K = 3.05$ microns. So, the difference is not that extreme and the lens might be permissible for the application.

See R.E. Fisher, B. Tadic-Galeb, P.R. Yoder: "Optical System Design", SPIE Press

1.2.6 Strehl ratio

$$\boxed{\mathrm{Strehl} = \exp(-4\pi^2 W_{\mathrm{RMS}}^2) \approx 1 - 4\pi^2 W_{\mathrm{RMS}}^2 \geq 0.8}$$

W_{RMS} - RMS wavefront error in λ

Example: Compute for the aforementioned example with $W_{RMS} = 0.08\,\lambda$, the maximum energy at the center of the spot compared to a diffraction limited system.
Solution: $\mathrm{Strehl} = 1 - 4\pi \cdot 0.08^2 = 0.75$ %

See W.J. Smith: "Modern Optical Engineering", SPIE Press , chapter 11

1.2.7 RMS and PV wavefront error

$$\boxed{\mathrm{RMS} \approx \sqrt{\sum C_j^2} \leq 1/14}$$

$$\boxed{\mathbf{W_{PV}} \leq \lambda/\mathbf{4}}$$

W_{RMS} - RMS wavefront error in λ
C_j - Zernike coefficients given in lambda
λ - wavelength

You have an optical system with an F-number of 4 that is described by Zernike coefficients for different field points. The dominant aberrations are spherical aberration and coma with Spherical: 0.05, Coma: 0.07. How about the Strehl and the radius for 80% encircled energy for your system?

Solution: Lets assume that Coma in x and y direction are given. So we have 3 Zernikes and we get $RMS = \sqrt{0.05^2 + 0.07^2 + 0.07^2} = 0.11$.

Therefore Strehl becomes 52% and the spotdiameter for encircled energy becomes 9.2 microns (compared to an airy diameter of 4.9 microns).

Compare this with the example having 0.08 RMS. Obviously, the Strehl very strongly depends on the wavefront error.

Beware: Of course all such estimations are 1) only rough estimates and 2) only OK if the aberrations are quite small. If you need a better estimate: Use an optical simulation tool (e.g. Zemax).

1.3 Noise

1.3.1 Averaging

$$\boxed{\Delta N = \sqrt{N}}$$

N - number of independent measurements
ΔN - statistical improvement factor due to averaging

Example: You have to measure the distance between two lines. For the position resolution we have a repeatability of 10 microns. The lines are 100 Pixels long. How accurate can we determine the separation of the two lines.
Solution: $\sqrt{100} = 10$ so we improve by a factor of 10 and therefore the repeatability is 1 micron.

Beware: For the measurement uncertainty (which is often what we are really interested in) $\sqrt{N}$ works of course only for the statistical part of the error. Systematic errors are not taken into account. They will be unaffected by the averaging.

See Wikipedia: Normal Distribution

1.3.2 Photons

$$\mathbf{E} = \mathbf{h}\nu = \frac{\mathbf{h}\,\mathbf{c}}{\lambda} = \frac{\mathbf{198.8 \cdot 10^{-27}}}{\lambda}\,\mathrm{J\,m}$$

E - energy per photon
h - Planck's constant $= 6.626 \cdot 10^{-34}$ J s
c - speed of light $= 3 \cdot 10^{8}$ m/s
λ - wavelength

Example: We have a light source with 1 mW at $\lambda = 500$ nm. How much photons do we get in 10 seconds ?

Solution: The energy of 1 photon is $E = 4.0 \cdot 10^{-19}$ J. 1 mW in 10 seconds means that we have a total energy of 10 mJ and therefore we end up with $2.5 \cdot 10^{16}$ photons.

See Wikipedia: Photon

1.3.3 Signal to noise ratio

$$\mathrm{SNR} = \frac{\mathrm{P\ QE\ t}}{\sqrt{(\mathrm{P}+\mathrm{B})\ \mathrm{QE}\cdot\mathrm{t}+\mathrm{DC\ t}+\mathrm{RON}^2}}$$

SNR - signal to noise ratio

P - optical power per pixel in photons per second

QE - quantum efficiency of imager

t - exposure time

B - background illumination per pixel in photons per second

DC - dark current in electrons per s at the given temperature

RON - read out noise per pixel in electrons

The terms under the squre root in the denominator are quadratically summed noise contributions. For the dark current: The dark current itself is given by $DC \cdot t$. The variation (standard deviation) of the dark current is $\sqrt{DC \cdot t}$. This squared gives again $DC \cdot t$. The same principle applies for the photon noise. For the read-out noise the specified numbers are already given as a standard deviation. Therefore, the additional square operation is necessary.

Example: You use an imager with a quantum efficiency of 0.7 at your working wavelength (500 nm). Background illumination is negligible and approximately 10 nW fall onto the senor with 1 million pixels. Read-out noise of your sensor is 4 electrons and dark current is 20 electrons per second per pixel (might be also a binned pixel). Our exposure time is 0.1 seconds. Compute the SNR that we will achieve.

See PCO White paper SNR, pco_cooKe_kb_snr_0504.pdf, www.pco.de

Solution: The power P per pixel is 10 nW / 1.000.000. Divided by the energy per photon (see last section) we get P=25.100 photons per pixel per second. Therefore,

$$SNR = \frac{25100 \cdot 0.7 \cdot 0.1}{\sqrt{25100 \cdot 0.7 \cdot 0.1 + 2 + 4^2}} = 42 \tag{2}$$

Compare this with a cheaper sensor (QE 0.5 and read-out noise of 10 electrons): Here the SNR drops to 34 (5 Bits and 30 dB).

1.3.4 Dynamic Range

$$\boxed{\text{Dynamic Range in dB} = 6N}$$

$$\boxed{\text{Dynamic Range} = \frac{\text{FWC}}{\text{noise}}}$$

N - number of (effective) bits of the imager
FWC - full well capacity of one pixel in electrons
noise - noise per pixel in electrons

Example: You have an expensive sensor with large pixels and the full well capacity is 100.000 electrons. The sum of the noise electrons per read-out for your application has been estimated to 5 which gives also something like the "smallest detectable signal". Compute a suitable number of bits per pixel for your application. Give the dynamic range in dB. Compare with a simple sensor (10.000 electrons, 20 electrons read-out noise).

Solution: The dynamic range is $100.000/5 = 20.000$. Therefore, we will need the logarithm dualis of 20.000 as the number of bits: $N = \text{ld } 20.000 \approx 14.2$ bits. The dynamic range in dB is approximately $6 \cdot 14.2 \approx 85$ dB.

For the simple sensor: $10.000/20 = 500$, 9 Bit, 54 dB.

See R.D. Fiete: "Modelling the Imaging Chain of Digital Cameras", SPIE Press

1.4 Misc

1.4.1 Power within a spot

$$\boxed{\text{AiryPower } \mathrm{P}(\mathrm{r} < \mathrm{r_A}) \rightarrow 84\%}$$

$$\boxed{\text{AiryPower } \mathrm{P}(\mathrm{r} < 2\mathrm{r_A}) \rightarrow 91\%}$$

$$\boxed{\mathbf{PL} = \exp(-\mathbf{2}\frac{\mathbf{D_{EP}}}{\mathbf{D_{1/e^2}}})}$$

PL - powerloss due to clipping of the Gaussian beam
D_{EP} - diameter of entrance pupil
D_{1/e^2} - $1/e^2$ diameter of Gauss
r_a - resolution according to Airy $(1.22\lambda K)$

Example: We use a laser with a diameter of 1 mm (defined using $1/e^2$). The size of our entrance pupil is 1.5 mm. Compute the loss of power.

Solution:

$$\exp(-2 \cdot 1.5/1) \approx 5\% \tag{3}$$

See Melles Griot Technical Guide, cvimellesgriot.com Gaussian Beams

1.4.2 Coherence length and average speckle size

$$\boxed{\mathbf{L} = \frac{\lambda^2}{\boldsymbol{\Delta}\lambda}}$$

$$\boxed{\mathbf{r_{aver}} \approx \mathbf{0.61}\frac{\lambda}{\mathbf{NA'}}}$$

L - coherence length

$\Delta\lambda$ - full width half maximum bandwidth of light source

λ - wavelength of light source

r_{aver} - average speckle radius.

NA' - image sided numerical aperture

Example: : You are using a laser diode at 600 nm with a FWHM of 0.1 nm. The object has a roughness of 50 nm. Estimate if the surface roughness is small enough, so that we can neglect speckles.

Solution: The coherence length is $L = 600^2/1 = 360.000$ nm $= 0.36$ mm. So it is considerably above the surface roughness. In order to see if we have "enough" roughness for speckles we also have to check if the roughness is approximately larger than $\lambda/20 = 30$ nm which is the case here.

Example: We do not want to be bothered much by the speckles so we decide that on average 25 speckles should fall onto one pixel of our sensor (having 8 microns pixel size). What F-number are we allowed to use ?

See Wikipedia: Speckle Pattern

Solution: 25 speckles means that 5 x 5 speckles will fall onto one pixel. Therefore, the average speckle size should be 8 microns / 5 = 1.3 microns.

For the speckle radius this means that it should be smaller than 0.65 microns. Therefore:

$$0.65 = 0.61\lambda/NA \tag{4}$$

and so obtain $NA \approx 1$ and therefore the F-number would be 0.5. This is not realistic. Therefore, we cannot achieve this goal.

1.5 Confidence, true positives, false negatives etc.

1.5.1 Bayes probability

$$\boxed{\mathbf{P_{A|B} = \frac{P_{B|A}P_A}{P_{B|\neg A}(1 - P_A) + P_{B|A}P_A}}}$$

$$\boxed{\text{sometimes :}\mathbf{P_{B|\neg A} = 1 - P_{B|A}}}$$

P_A - probability that A is true
P_B - probability that B is true
$P_{A|B}$ - probability that A is true given that B is true
$P_{B|A}$ - probability that B is true given that A is true
$P_{B|\neg A}$ probability that B is true given that A is NOT true

Example: Suppose that your image processing system finds a faulty part with 99% probability. The production of the parts is quite good and only 2% of the parts are really faulty. Lets say your system detected a faulty part. How large is the probability that this part is really faulty and how large is the probability that it is not faulty at all?

Solution: B denotes the event that we detected "faulty". A denotes the event that the part *is* faulty. We are interested therefore in $P_{A|B}$.

See Wikipedia: Bayes Theorem

$P_A = 0.02$
$P_{B|A} = 0.99$ is the probability that IF A is faulty, our system indeed says that it is faulty. The only thing that is missing is $P_{B|\neg A}$ and we can assume here: $P_{B|\neg A} = 1 - P_{B|A}$ because there are only two options available for B to occur. Namely A is true or not true and therefore: $P_B = P_{B|A} + P_{B|\neg A}$

But: We are sure that B has occurred ! P_B is now for us a certainty because we started with the fact that our system HAS detected something so we can write

$$P_{B|\neg A} = 1 - P_{B|A} = 1 - 0.99 = 0.01 \tag{5}$$

Therefore:

$$\begin{aligned} P_{A|B} &= \frac{0.99 \cdot 0.02}{(1 - P_{B|A})(1 - P_A) + 0.99 \cdot 0.05} && (6) \\ &= \frac{0.99 \cdot 0.02}{0.01 \cdot 0.95 + 0.99 \cdot 0.02} = 0.67 && (7) \end{aligned}$$

This means that the probability that this part which has been detected as "faulty" really is faulty in "only" 67% of all cases. Or in other words: 33% of the parts being marked as faulty indeed are not faulty at all.

This might be counter-intuitive and humans are very bad at estimating such things. The lower the probability of an error, the more counter intuitive is the result. Therefore, a concrete computation most of the time is necessary.

This kind of reasoning becomes especially important when fulfilling a given standard of quality (typically customers are not happy if you wrongly label 33% of the production as being flawed). If the quality standard is very high, then the image processing inspection must be classified correctly with an extremely low error !

1.5.2 Uncertainty

$\text{PV} \approx 6..10 \cdot \text{RMS}$

$\text{Uncertainty} \approx 10 \cdot \text{Resolution}$

$1 \cdot \sigma \rightarrow 68\%$

$2 \cdot \sigma \rightarrow 95\%$

$3 \cdot \sigma \rightarrow 99.7\%$

Example: Your measurement system should measure the distance to a target. The measurement result should be correct in 95% of all measurements. Assume that the errors vary statistically due to noise of the CCD. The final measurement uncertainty should be 10 microns. What measurement uncertainty specified at 1 sigma do you allow? What resolution should your system have ?

Solution: Based on the 95% we estimate that we need 2 σ and therefore we conclude that the measurement uncertainty at 1σ should be $\sigma = 5$ microns. The resolution should be about 0.5 microns.

Example: You are measuring the topography of a surface. The PV error of the measurement is 10 microns. Estimate the RMS error of the measurement.

Solution: $10\ /\ 6 = 1.6$ microns.

See Wikipedia: Normal Distribution

1.5.3 Interference filter

$$\boxed{\lambda_{\mathbf{phi}} = \lambda\sqrt{\mathbf{1} - \frac{\sin^2\phi}{\mathbf{n}^2}}}$$

λ_{phi} - filter center wavelength when filter is rotated

ϕ - rotation angle

n - effective index of refraction of material (1.5 ... 1.9)

λ - center wavelength of filter

Example: You have an interference bandpass filter that is specified for $\lambda = 500$ nm. However, you need a bandpass at 510 nm. How much should you rotate the filter?

Solution: We just assume $n = 1.5$ and solve the equation for ϕ

$$\phi = \arcsin(n \cdot \sqrt{1 - (500/510)^2}) = 17^\circ \tag{8}$$

See W.J. Smith: "Modern Optical Engineering", SPIE Press , chapter 7

1.6 Photometry and radiometry

Things are getting, unfortunately, a little bit uncomfortable here because every quantity we are dealing with now comes in two "flavors": Photometric and radiometric. Typically, the photometric quantity X is denoted by X_e in radiometry but sometimes also X is used for the radiometric quantity and photometry is then indicated by X_λ. To make things more complicated, we, of course, need different names and different units for these quantities and — even worse — due to historical reason, for one and the same thing different units are often used.

This is not a textbook about photometry and radiometry and we try to make things as simple as possible. Even by risking that we will be a little bit "unclean". All the equations are written without any index. In addition, e.g. $E = \Phi/A$ is a more or less crude approximation for $E = d\Phi/dA$. This means that the equation is only valid in the infinitesimal sense.

For industrial image processing, my advice is to keep things as simple as possible by working completely in radiometric units and to restrict yourself to a pseudo-monochromatic case (one dominant wavelength). Convert at the beginning or the end from photometric units into radiometric ones (if necessary). Otherwise things are getting soon too complicated to be done fast by hand.

1.6.1 Irradiance and illuminance

$$\boxed{\mathbf{E} = \frac{\mathbf{\Phi}}{\mathbf{A}}}$$

E - irradiance or illuminance

Φ - flux or illuminous flux

A - area

Example: Compute the illuminance on your CCD chip (2/3 inch) if you have a (luminous) flux of 0.001 lumens

Solution: $E = 0.001/A$ with $A = 8.8 \cdot 6.6$ mm^2 gives $E = 17.2$ lx.

See A.V. Arecchi, T. Messadi, R.J. Koshel, "Field Guide to Illumination", SPIE Press Radiometrie

1.6.2 Conversion between radiometry and photometry

$$\boldsymbol{\Phi}(\lambda) = \mathbf{683}\,\frac{\mathrm{lm}}{\mathrm{W}} \int_0^\infty \mathrm{V}(\lambda)\,\Phi_\mathrm{e}(\lambda)\,\mathrm{d}\lambda$$

Φ - a photometric quantity, e.g. flux
Φ_e - a radiometric quantity, e.g. flux
λ - wavelength
V - normalized sensitivity of the eye

Example: The illuminance on your sensor is E=0.5 lx. You are using red light at a wavelength of approx. 600 nm. Compute the irradiance.

Solution: For monochromatic light the equation becomes

$$\Phi(\lambda) = 683\,\frac{\mathrm{lm}}{\mathrm{W}}\mathrm{V}(\lambda)\Phi \tag{9}$$

and $V(600\text{ nm})$ — according to the table on the right-hand side of the wallpaper — is 0.631.

Therefore we get: $\Phi_e = 0.5/683/0.631 = 1.1\ \mathrm{mW/m^2}$.

See A.V. Arecchi, T. Messadi, R.J. Koshel, "Field Guide to Illumination", SPIE Press

1.6.3 Radiant and luminous intensity

$$\boxed{\mathbf{I} = \frac{\mathbf{\Phi}}{\mathbf{\Omega}}}$$

$$\boxed{\mathbf{\Omega} = \frac{\mathbf{A}}{\mathbf{r}^2} \approx \pi \sin^2 \alpha}$$

I - radiant or luminous intensity

Φ - flux or illuminous flux

Ω - solid angle

A - area

r - distance from area to the source

α - half angle of the angle extended by area to source

Example: Your light source has a luminous intensity of 100 candela and illuminates your object at a distance of 1 m. The luminous intensity is more or less constant over the angle. Compute the illuminance of the surface.

Solution: The solid angle is given by $\Omega = A/1^2 = A$ sr/m^2. With $I = \Phi/\Omega$ we get $\Phi = I \cdot A$ and therefore $E = \Phi/A = I \cdot lx/cd = 100$ lx.

See A.V. Arecchi, T. Messadi, R.J. Koshel, "Field Guide to Illumination", SPIE Press

1.6.4 Radiance and luminance

$$\boxed{\mathbf{L} = \frac{\boldsymbol{\Phi}}{\mathbf{A}\,\boldsymbol{\Omega}}}$$

L - radiance or luminance

Φ - flux or illuminous flux

A - area

Example: The "brightness" (just another word for luminance (and sometimes even for radiance)) of your illuminated object is 50 W/(m^2 sr). You assume again a homogeneous light emission and the area of the object is 10 x 10 mm. Compute the flux (power) that the object emits towards the pupil of your camera which is located at a distance of 1 m and has a diameter of 10 mm.

Solution: With $\Omega = A/r^2 = \pi r^2/1^2 = 0.0079$ we get $\Phi = L \cdot 0.01^2 \cdot \Omega = 39\ \mu$W

See A.V. Arecchi, T. Messadi, R.J. Koshel, "Field Guide to Illumination", SPIE Press

1.6.5 Irradiance and illuminance

$$\boxed{\mathbf{E} = \pi \mathbf{L} \sin^2(\alpha') \cos^4(\omega')}$$

E - irradiance or illuminance

L - radiance or luminance

$\sin\alpha'$ - image sided numerical aperture

ω' - (image-sided) field angle

Example: Your object emits light (after being illuminated) with a radiance of 10 $W/m^2/sr$. Your camera lens has an F-number of K=5. Compute the on-axis irradiance on your CCD chip.

Solution: K=5 corresponds to a numerical aperture of $\sin\alpha' = 1/(2K) = 0.1$. On-axis means that the field angle is zero. Therefore, $E = \pi \cdot 10 \cdot 0.1^2 = 0.314$ W/m^2.

Example: You use a lens with a focal length of 20 mm with your 1/3 inch sensor. How strong is the natural vignetting due to the lens?

Solution: The sensor has a diagonal of 6 mm. So the maximum image-sided field angle is $\tan\omega' = 3/20$ and therefore $\omega' = 0.15$ rad. With that the natural vignetting becomes $\cos^4\omega' \approx 0.95$ which means that at the edge of your sensor the irradiance is reduced by 5%.

See A.V. Arecchi, T. Messadi, R.J. Koshel, "Field Guide to Illumination", SPIE Press

1.6.6 Lambertian surface

$$\boxed{\mathbf{I}(\phi) = \mathbf{I_0}\cos(\phi)}$$

$$\boxed{\mathbf{E} = \pi\mathbf{L}}$$

I_0 - radiant or luminous intensity on axis
$I(\phi)$ - radiant or luminous intensity in direction ϕ
L - radiance or luminance
E - irradiance or illuminance

Example: You have a very diffuse scattering surface with an area of 0.5 square meters which is illuminated with an irradiance of 1 W/m^2. 50% of the light will be absorbed. Compute the radiant intensity in the direction of 20° to the surface normal.

Solution: $I = L \cdot A = E \cdot 0.5/\pi \cdot 0.5 = 0.079$ W/sr on axis and multiplied with the cos(20) this gives I(20°) = 0.075 W/sr

See A.V. Arecchi, T. Messadi, R.J. Koshel, "Field Guide to Illumination", SPIE Press

1.6.7 Irradiance on object

$$\boxed{\mathbf{E_{obj}} = \frac{\pi}{\mathbf{80.000}} \frac{(\mathbf{a^2} + \mathbf{y^2}) \cdot \mathbf{N^4} \cdot \mathbf{ps^2} \cdot \delta\mathbf{z^2}}{\mathbf{a^4}\,\mathbf{S} \cdot \mathbf{y^4} \cdot \mathbf{t}}}$$

E - irradiance or illuminance on object

t - exposure time

a - object distance

y - object field size

N - number of camera pixels in one direction

S - sensitivity of camera (in nJ per pixel)

z - required depth of field in object space

ps - pixelsize

This equation is a combination of lots of basic equations in order to compute the irradiance on the object. It assumes a Lambertian surface and all losses due to the optics (reflections, filters) or the object (absorption) are not considered (because you can simply consider them at the end your computation by a simple multiplication).

Example: You use a good camera (Dalsa Pantera) for imaging an object (100 x 100 mm) moving on a conveyor belt. Due to the movement, the exposure time should be short (1 ms). You are using 1000 x 1000 pixels with a pixelsize of 12 x 12 microns. The working distance is 1 m. The required depth of field is 5 mm. Compute the necessary irradiance on the object.

Solution:

$$E_e = \frac{\pi}{80000} \frac{(1^2 + 0.1^2)1000^4 \cdot (12 \cdot 10^{-6})^2 0.005^2}{1^4 \cdot 20 \cdot 0.1^4 \cdot 0.001} = 0.071\,\mathrm{W/m^2} \tag{10}$$

See Appendix

1.6.8 Scattering surface

$$\boxed{\mathbf{\frac{R_{scatter}}{R_{total}} = 1 - \exp(-(4\pi \cos\phi_{in} \cdot \frac{\sigma_{RMS}}{\lambda})^2)}}$$

$R_{scatter}/R_{total}$ - ratio of scattered light to total reflected light

ϕ_{in} - incidence angle

σ_{RMS} - RMS surface roughness

λ - wavelength

Example: A surface under test is illuminated and everything is set up correctly so that you have the right irradiance on your sensor when a white sheet of paper is imaged. Now the object is a nicely polished sheet of aluminium with a surface roughness of 20 nm RMS. The reflected part of the light will miss the camera. By what factor should you increase the flux of your lamp ($\lambda = 500$ nm) ?

Solution: According to the table on the right-hand side we find that polished aluminium has about ($0.65/0.7 \approx 1$) the same reflectivity. But whereas for paper nearly all of the totally reflected light will be diffusely reflected, for the aluminium we only have

$$\frac{R_{scatter}}{R_{total}} = 1 - \exp(-(4\pi\cos(0) \cdot \frac{20}{500})^2 = 0.22 \tag{11}$$

Therefore you should increase the power of the lamp by a factor of 5.

See Harvey et al., Optical Engineering 51, pg. 013402, 2011

1.6.9 Spectral filtering

$$\boxed{\mathbf{\Phi_e[W] = (\lambda[\mu m] - 0.4)\frac{FWHM[nm]}{10}} \quad @\,3000\,\mathrm{K},\ 1000\ \mathrm{lm}}$$

$$\boxed{\mathbf{\Phi_e[W] = FWHM[nm] \cdot 0.02} \quad @\,6000\,\mathrm{K},\ 1000\ \mathrm{lm}}$$

FWHM - full width half max of spectral filter in nm
Φ_e - flux in W that passes through filter given a source with 1000 lm and a given color temperature (3000 or 6000 K)
λ - filter central wavelength

Example: You measured sunlight at the object with an illuminance of 10.000 lux. In order to reduce the disturbing (because not constant) sun light you use a LED illumination with 10 nm FWHM and a corresponding filter. How strong should the artificial (LED) irradiance on the object be chosen if you want to have the sun radiance below 10% of the LED illumination ?

Solution: We compute the maximum irradiance due to the sun (T=6000 K): $\Phi_e = 10 \cdot 0.02 = 0.2$ W/m^2. This means that the LED illumination system should deliver at least 2 W/m^2.

You might compare that with the values on the right-hand sided table on the wallpaper for different illuminance levels.

See Appendix

2 Some more extended examples

2.1 Standard 2D imaging

Your task is to inspect 100 mm cogwheels for scratches (width $\geq$ 100 microns). The working distance should be 1 m and you hope that you can use a 1/3 inch sensor. The objects are delivered by a conveyor belt with a speed of 0.5 m/s and the axial distance varies by +- 10 mm.

Solution:

We start by computing the magnification and the focal length of the lens.

From the table (right hand side of the wallpaper) we find that the sensor size is 4.8 x 3.6 mm. Therefore the magnification β' (top of the wallpaper) is

$$\beta' = M = \frac{a'}{a} = \frac{y'}{y} = \frac{-3.6}{100} = -0.036 \tag{12}$$

Beware: A typical error during computation is that you forget to use the correct sign conventions (or you use different sign conventions). For this wallpaper we use the most often used convention that all variables in image space have a prime and that the origin of the coordinate system is the lens (more accurate: The principal planes of the lens).

As a rough estimate we could take for the focal length f'.

$$f' \approx a' \tag{13}$$

and with Eq. 12 we have

$$f' = \beta' \cdot a = -0.036 \cdot -1000 = 36\,\text{mm} \tag{14}$$

A more accurate computation would involve

$$\frac{1}{a'} = \frac{1}{a} + \frac{1}{f'} \tag{15}$$

which leads to

$$f' = 34.75\,\text{mm} \tag{16}$$

Typically, we will anyway not get the exact focal length (because we neither want to use zoom lenses (because of the unwanted change in magnification that might be introduced by mechanical vibration) nor our own lens design (too expensive)). Therefore, in most cases (with $|\beta'| < 0.1$) the approximation of Eq. 13 is more than good enough.

When choosing the appropriate available lens, we have to chose the shorter focal length. This means that if we only could chose between $f' = 40$ mm or $f' = 30$ mm, we would have to take the 30 mm lens because otherwise we would not see the whole cogwheel.

In the following we assume that we have a 35 mm lens available.

We now have to think about the necessary resolution. This is a quite complicated topic because it depends a lot on what we really want to achieve. An exact specification of the task is absolutely mandatory.

Lets make things simple here. We will assume that we only have to *detect* the scratches. We might use dark field illumination (see bottom left part of the wallpaper). It strongly depends now on the roughness and structure of the object. Here, we might assume that it is enough to image the width of the scratch onto 1 pixel. Since the scratch has a certain width, we have some sort of "averaging" effect which will help us to detect even such 1-pixel-small scratches. For polished surfaces (e.g. wafers) we might even detect much smaller scratches.

Therefore, we assume that 1 pixel should correspond to 100 microns:

$$\textit{pixelsize} \leq 0.1 \cdot \beta' = 3.6\,\text{microns} \tag{17}$$

For our sensor this means that we need at least

$$\frac{4.8}{0.0036} \cdot \frac{3.6}{0.0036} = 1.3\,\text{MPixel} \tag{18}$$

We continue with the F-number K of the lens. The F-number is basically important for three different important parameters: 1) the optical resolution, 2) the depth of focus, and 3) the amount of light that we capture.

In this section we do not care about radiometry and we will, therefore, neglect issue 3. We first concentrate on the depth of focus. We can use the simple equation at the top of the wallpaper

$$\delta z = \frac{2KB'}{\beta'^2}, \tag{19}$$

where B' denotes the diameter of the allowed blur circle in the image plane. Due to the conveyor belt we have $\delta z = 50$ mm.

Eq. 19 gives us

$$K \leq 9 \tag{20}$$

This is a limitation concerning light efficiency so we already see that we will need plenty of light for this application.

But it also helps us thinking about the resolution. K=9 means that the lens is stopped down strong enough so that aberrations will not be our limitation concerning the lateral resolution of the imaging. Instead we can use the Airy resolution (top of the wallpaper):

$$r_A = 1.22\lambda K = 6\,\text{microns} \tag{21}$$

This might indeed be a problem because it means that the theoretical resolution is nearly twice the pixel size.

Do we really care? No. Because for this application it is not really necessary that the lateral resolution equals the critical dimension (width of the smallest scratches). We do not need the Airy two-point resolution.

If we would need a better lateral resolution at this point, we now would have to redesign our application.

We only want to search for scratches we, therefore, might use a non-telecentric lens.

2.2 A variation: Measure the diameter

Now, let us say that in addition we also want to measure the diameter of some holes in the cogwheel. Again, we have to think about the resolution. And again, we do not really need the two-point resolution. We are interested in the average of the position resolution. For good conditions we might assume a position resolution of

$$r_{pos} \approx \max(r_A, ps)/5 = 1.2\,\text{microns} \tag{22}$$

in image space and, therefore, $r_{pos}/\beta' = 0.034$ mm in object space. If we can average over the circumference of the hole consisting of N pixels, we might gain an additional factor $\sqrt{N}$ which will lead to a very precise measurement of the hole diameter.

But we have to keep in mind now the perspective of the imaging. We definitely want to use a telecentric imaging system. The best option would be to buy a good telecentric lens. If you want to use your own (cheaper) system: Build a simple telescope out of two objective lenses corrected for infinity. The magnification is given by

$$\beta' = -\frac{f_2}{f_1} \tag{23}$$

So, if we chose $f_1 = 300$ mm, $f_2 = 10.8$ mm follows.

2.3 Example with small structures

Assume that we want to image written text on the cogwheel. The letters are very small. We assume a critical dimension of 0.1 mm, e.g. (OK, this does not make much sense in this case, but if the object is much larger the relative resolution of 1:1000 can be realistic.)

How about the necessary resolution of the optics and the sensor ?

The table on the left-hand side gives us the necessary rules of thumb. For neighboring structures we need $r_a \approx 1$ CD $= 0.1$ mm. According to the Rayleigh resolution this would imply

$$0.1\,\mathrm{mm} \cdot \beta' = 1.22\lambda K \tag{24}$$

or $K \leq 5.3$ (which would lead to problems concerning the depth of field but let us forget about that).

We also would need 3..4 pixels for the critical dimension to have enough sampling. This leads to

$$0.1\,\mathrm{mm} \cdot \beta'/3.5 = \mathit{Pixelsize} \approx 1 \text{ micron} \tag{25}$$

Now, this definitely will lead to a problem because we will not have such small pixels.

For a 100 mm Object and a critical dimension of 0.1 mm we will need a sensor with something in the range of 3000 x 4000 pixels. It is possible to use such a sensor but in this case the sensor size will be larger than 1/3 inch (and of course more expensive).

Lets assume we take a 4/3 inch sensor. We rerun our computation (nice that we have it in octave, isn't it ?) and end up with a pixel size of 3.7 microns.

Now, we might have to care about the aberrations of the lens working at K=5.3. If we would have the spotsize of the lens over the field, everything would be easy to estimate. But typically we do not have that.

Let's assume that our image processing algorithms are good enough to handle the writing even if the contrast is only 10%. According to

$$MTF(f) \approx 1 - f = 0.1 \tag{26}$$

a we come to the normalized frequency f =0.9. When using the new $r_a = 21$ microns we obtain

$$f_{max} \approx \frac{1.22}{r_A} = \frac{1}{K\lambda} = 45\ lp/mm \tag{27}$$

So we desire the MTF to have a contrast of at least 0.1 even at 45 lp/mm over the whole field at K=5.3.

Comment: We used a Q of 3..4. This seems to be a quite large value if you compare it with what one uses in photography or satellite imagery. For this application it is beneficial to use such a strong (over-)sampling (compare the appendix).

2.4 How about the lighting ?

This all seems fine but how much light do we need ? In this case we will assume that the motion-induced blur should be less than 1 pixel. For a conveyor belt speed of $v = 1$ m/s this translates into

$$\delta y' = pixelsize = v \cdot t \times \beta' \tag{28}$$

and with that we obtain $t = 0.2$ ms.

This is a rather short exposure that we can use.

Now, the main question is: How much irradiance/illuminance do we need to finally have enough photons on the CCD?

For sure, this depends on a lot of factors. In practice, often not even simple estimates are performed but rather one buys a lamp just based on some sort of past experience. This might turn out to be a disaster.

(Since nobody seems to be able to perform photometric computations, most manufacturers of lighting equipment started to just give not even real specifications concerning the optical output of their sources!)

We at least want to estimate the necessary irradiance. For that purpose we first assume a completely Lambertian surface (perfectly diffuse). In a first approach we also neglect the absorption or the albedo of surface and losses due to optics.

With

$$E_{obj} = \frac{\pi}{80.000} \frac{(a^2 + y^2) \cdot N^4 \cdot ps^2 \cdot \delta z^2}{a^4\, S \cdot y^4 \cdot t} \tag{29}$$

we can estimate the necessary irradiance on the object. N denotes our number of pixels of the sensor for the smaller side. Here we computed $N = 3000$. δz is the depth of field which is 20 mm, y is our field size (100 mm) and S denotes the responsitivity of the sensor in units of nJ/cm^2. As a standard value (see wallpaper right table) we take 100 nJ/cm^2. This way we arrive at 678 W/m^2.

Now we include albedo and the optics. For the optics we assume a 30% light loss (of course strongly depending on the lens) and for the reflectance we take "Unpolished Aluminium" from the table (wallpaper right side) R=0.55.

This gives us a factor of

$$F = 0.7 \times 0.55 = 0.38 \tag{30}$$

And therefore we need about 1762 W per square meter. This is a lot. Bright sun light reaches about 1000 W/m^2 but within a narrow spectral region it is much less !

If you like you might change to photometric units. If we would assume that the source emits yellow-green light, we would need

$$E = 683 \frac{\mathrm{lm}}{\mathrm{W}} \cdot E_e = 683 \frac{\mathrm{lm}}{\mathrm{W}} \cdot 1762 W \approx 1.200.000 \text{ lx} \tag{31}$$

What happens if we emit light at $\lambda = 450$ nm ? Of course, the radiometric result will not change (if we assume that the sensitivity of the CCD is also 100 nJ per square centimeter at that wavelength) but necessary amount of illuminance reduces.

You might think that the whole thing does not make any sense because our plan was to use dark field illumination, so nearly all of the light is lost. But this is a misconception. Of course, most of the CCD pixels will not receive (much) light and, therefore, lots of light is lost. But we are only interested in the irradiance of the pixels that image the scratches. Of course, it is a very strong assumption to say that they will scatter according to a Lambertian model and of course this is wrong. But at least it gives us an estimate (might be wrong by a factor of ten but better than nothing at all).

The details of light scattering at a small scratch are so complicated and depend so strongly on the actual scratch geometry that we anyway can only hope to have a rough estimate. Only detailed measurements at a lot of realistic scratches would give us a better estimate.

2.5 Additional light

Assume that you have an application where, unfortunately, additional light which is not under our control falls onto the object. Let's assume that from time to time direct sunlight falls onto our object. The sensor as well as our algorithms might have a problem with that and we might e.g. desire that the variation of lighting due to this unwanted contributions is below 10% of the main lighting.

Under direct sunlight we might have up to 100.000 lx. If a shadow falls onto the object, this sunlight drops to about 10.000 lx. Anyway if you want to make sure that the variation of the additional light is below 10%, it immediately follows that our own illumination source should yield something like 90.000 $\cdot 10 \approx$ 1.000.000 lx.

Well, this is difficult. For most large objects it is perhaps not realistic at all.

A typical approach is to use a (nearly) monochromatic illumination and to block all other parts of the spectrum by using a filter in front of the camera (beware of aberrations !).

We assume that we find a bright narrow-band LED (array) with Full-Width-Half-Maximum FWHM of $\Delta\lambda$= 5 nm.

How much sunlight variation will pass through that filter ?

$$\begin{aligned} \Phi_e[W/m^2] &= FWHM[nm] \cdot 0.02 \cdot \Phi[lx]/1000 && (32) \\ &= = 0.02 \cdot 5 \cdot 90.000/1.000 = 0.9\,\mathrm{W/m^2} && (33) \end{aligned}$$

In the yellow-green spectral region ($\lambda \approx 570$ nm, therefore $V \approx 1$) this translates into

$$0.9 \cdot 683 = 614\,\mathrm{lx} \qquad (34)$$

And therefore our lamp only has to deliver at least 6.140 lx. Still a high but not unrealistic value. We might try out an even smaller bandwidth, e.g. by using a laser (and then running into problems due to the laser safety).

Perhaps you now just should think about building a box around your setup in order to remove the sunlight altogether because your software department tells you that 10% variation is anyway not acceptable for their algorithms.

3 Appendix

3.1 Resolution

Very often, the choice of resolutions in an imaging system for an application at hand is a problem. You will find lots of different opinions and recommendations. This is because the "best" choice strongly depends on the application at hand and what you really want to achieve.

You definitely have to differentiate between the different kinds of resolutions. Also it should be clear that resolution is NOT "measurement accuracy". In the following, we are talking about the lateral resolution with respect to the ability to detect neighboring structures (points) as being separate. E.g. if you want to be able to read a letter, it is necessary that you detect the geometric primitives of the letter. We take as an example the letter "F" but of course we could also talk about two neighboring scratches. The smallest separation on the object that we want to discern is the so-called "critical dimension" CD.

Obviously we need 1) a good enough optical resolution (given by the diffraction limited resolution in combination with aberrations) and 2) enough sampling.

Often one uses the parameter "Q" for describing the relation between these two major influences:

$$Q = \frac{r_A}{p} \tag{35}$$

with the pixel size p and the Airy resolution (radius of Airy disc) of r_A. Here, we assume that we have diffraction limited resolution. If the aberrations are not negligible, you might use the real spotsize (beware that then the parameter often is not called "Q").

Now, what Q to chose for a specific application is not so easy to say. The values that are given on the wallpaper are based on simulations of images with these parameters. Fig. 1 shows a typical run of simulations. For the simulation, we used a typical design of a good lens, namely a Double-Gaussian system with an F-number of 3. The lens (taken from the Zemax sample catalog) is optimized for a distant object and a field of view of 28 degrees and polychromatic light.

The lens is optimized for a large format. The diffraction limited point spread function radius (Airy) is 2.1 microns. But the simulation in Zemax shows that the real RMS spotradius is something like 11.5 microns.

Now, of course it depends on the concrete application and the algorithms that we use for the processing of the images that we want. We assume that we want to clearly differentiate between the different bars of the "F" so this is our critical dimension.

According to the wallpaper we now would take something like $CD = r/0.7$ for the minimal critical dimension (in image space). Beware: Since we are far from the diffraction limit we have to use the real RMS spotradius instead of the Airy radius. Therefore, $CD = 11.5$ microns / 0.7 = 16.5 microns. Also according to the table, we should have between 3 and 4 pixels per critical dimension and this leads to a pixelsize of about 4 or 5 microns.

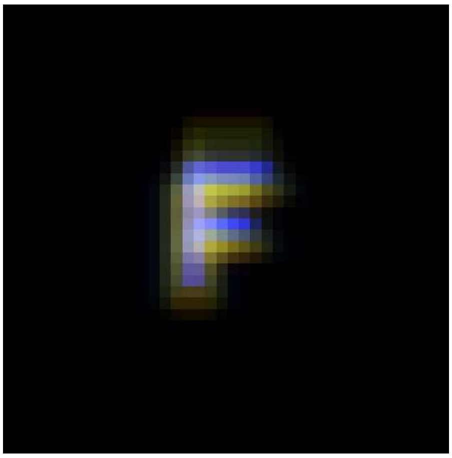
Fig 1 shows the simulated image on the sensor (no special care for Bayer mask) at different pixelsizes. Indeed: If we really want to resolve the details of the F, we need something between 4 and 5 microns. If we would chose CD = 2 pixel, the sampling would not be good enough. More sampling (more than 4 pixels per CD) is not necessary.

You might also use a different Q. Typically, this is done for most non-image-processing applications. But Q=1 already gives a result where you are not able to really resolve the "F".

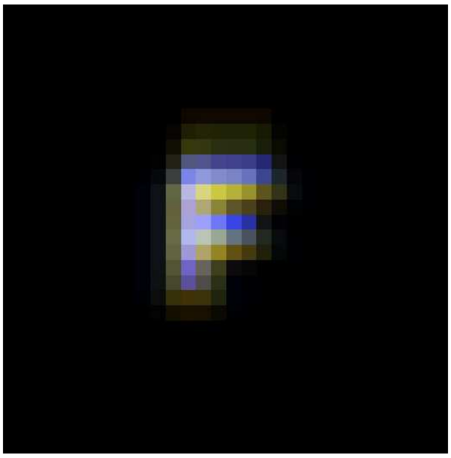
Nothing changes if we scale the lens for a smaller sensor. We definitely need a lot of pixels. This is a typical example where we want to have plenty of empty magnification. Most video microscopy systems are designed that way. It is not enough to just fulfill the sampling theorem. Rather we clearly oversample. By this approach the MTF of the sensor (pixels) gets high at the high frequencies and, therefore, we will still have an effective advantage for imaging. Also, noise tolerance is much improved.

So, the typical approach, that it is not necessary to have a better sensor resolution than the optical resolution, does not always makes sense. For photography and most imaging situations, yes. But for the inspection of very small structures pixel, oversampling is often a good idea.

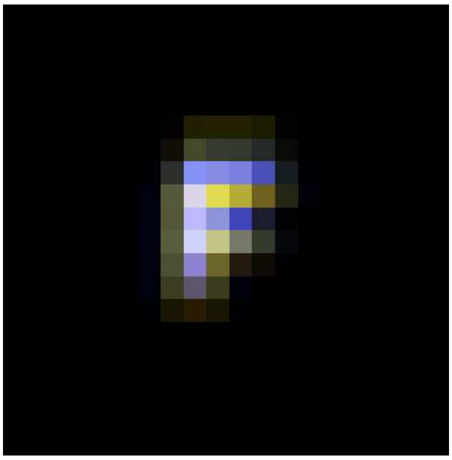
Unfortunately, there are costs involved because if we increase the number of pixels in one dimension by a factor of 2 we will end up having four times more pixels per image (resulting in increased cost and reduced speed). If possible, it is therefore recommended to perform a simulation of the complete imaging and then to decide about the resolutions (optical and sensor resolution) because obviously things are quite complicated.



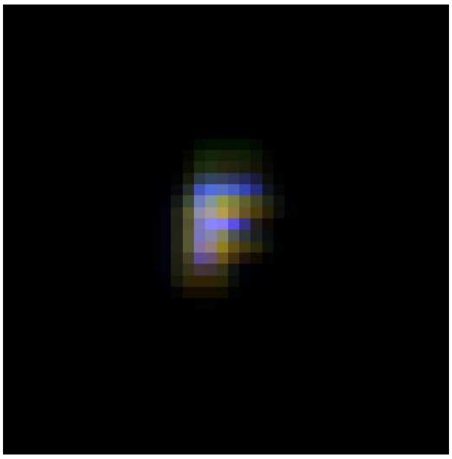
(a) 4 microns pixels (4 pixels per CD), Q=0.7



(b) 5 microns pixels (3 pixels per CD), Q=0.7



(c) 8 microns pixels (2 pixels per CD), Q=0.7



(d) 4 microns pixels (4 pixels per CD), Q=1

Figure 1: Simulation of a Double-Gauss type 28 degree lens, polychromatic light, RMS spotradius approximately 11.5 microns

3.2 Illumination

The equation,

$$E_{obj} = \frac{\pi}{80.000} \frac{(a^2 + y^2) \cdot N^4 \cdot ps^2 \cdot \delta z^2}{a^4\, S \cdot y^4 \cdot t}$$

estimates the necessary irradiance on the object for a given sensitivity and exposure time if we assume that the object is a perfect Lambertian white surface.

The basic idea to derive this equation is as follows:

The magnification is

$$\beta' = \frac{ps \cdot N}{\mathit{field}}, \tag{36}$$

where we denoted the pixelsize by ps, the number of pixels in one direction of the sensor by N and the extension of the object field by *field*.

The necessary irradiance on the CCD to achieve a level of 255 is

$$E_{CCD} = \frac{255}{S \cdot \tau} \tag{37}$$

if we finally want to use sensitivities in nJ per square centimeter, we have to multiply this by $10^-9 \cdot 10000$:

$$E'_{CCD} = \frac{255}{S \cdot \tau} \cdot 10^{-9} \cdot 10000 \tag{38}$$

Based on the required depth of focus δz we compute the F-number:

$$K = \beta'^2 \frac{\delta z}{2ps} \tag{39}$$

The focal length is given by

$$f = \beta' \cdot a \tag{40}$$

and with that we can compute the maximum field angle

$$\omega = \arctan \frac{field}{2a} \tag{41}$$

We can solve

$$E_{CCD} = \pi L N A^2 \cos^4 \omega \tag{42}$$

for L

and use that for a Lambertian surface

$$E_o = \pi L \tag{43}$$

Putting it all together gives us the desired equation.

3.3 Illumination 2

Two other equations are useful for estimating the amount of light within a spectral band:

$$\Phi_e[W] = (\lambda[\mu m] - 0.4)\frac{FWHM[nm]}{10} \ \ @\,3000\,\mathrm{K},\ 1000\ \mathrm{lm}$$

$$\Phi_e[W] = FWHM[nm] \cdot 0.02 \ \ l@\,6000\,\mathrm{K},\ 1000\ \mathrm{lm}$$

You typically need that if a spectral filter blocks part of unwanted light.

We are interested here only in thermal light (e.g. halogen lamps or the sun). Therefore, we start with Planck's law (see e.g. Wikipedia). You can forget about the normalization of the Planck law because we anyway do not know the absolute emission of the lamp. We first have to get the normalization right based on the luminous flux that the lamp emits. For the computation, we will assume a lamp with a luminous flux of 1000 lm. With the sensitivity of the eye $V(\lambda)$ we therefore have

$$\Phi = 1000\,\mathrm{lm} = 683\mathrm{lm/W} \cdot \int_{370}^{780} p_o\, Planck(\lambda) V(\lambda) d\lambda \tag{44}$$

The desired normalization constant is denoted by p_o.

(Planck gives something in radiometric units and by multiplying with V and the conversion constant 683 lm/W we get to photometric units.)

Based on this equation we can now easily obtain p_o:

$$p_o = \frac{1000}{683 \cdot \int_{370}^{780} \ Planck(\lambda)V(\lambda)d\lambda} \tag{45}$$

Now we are in the good situation to compute the amount of flux or luminous flux for other situations. We are interested here in the flux and we can directly write

$$\Phi_e = p_o \int_0^{\infty} \ Planck(\lambda)W(\lambda)d\lambda \tag{46}$$

where we denote the effective spectral window by $W(\lambda)$.

If we assume a Gaussian shaped spectral window with full-width-half maximum of *fwhm*, we can write

$$W(\lambda) = \exp(\frac{-4\log 2(\lambda - \lambda_0)^2}{fwhm^2}) \tag{47}$$

and now we have everything for numerically computing the flux for a given Gaussian shaped filter at a given color temperature for a 1000 lumen source (of course for a 2000 lumen source you just multiply the curves by two). This leads to a plot like the one depicted in Fig. 2:

We can make a linear approximation (to have a simple equation) and this leads to the two equations at the beginning of this section.

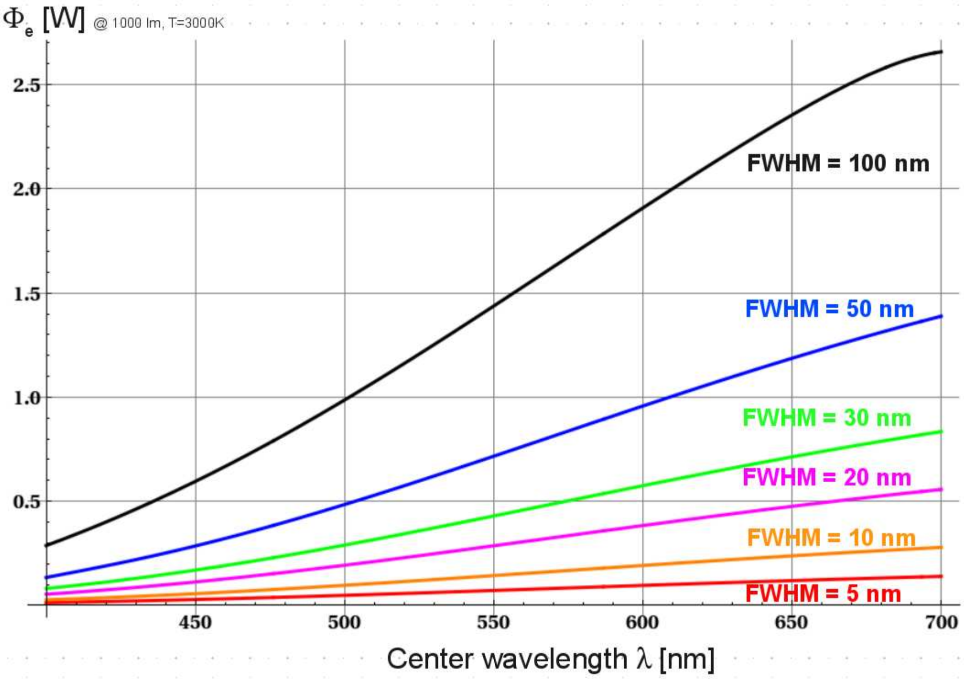


Figure 2: Simulation of the flux for a 1000 lumen source for 3000 K color temperature and with different Gaussian filters (horizontal axis gives the filter central wavelength λ_0 and the different curves correspond to different widths of the filter passband).

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